Infinite Automata, Logics and Games

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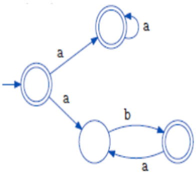
ω-Automata

Nondeterministic Tree Automata

A nondeterministic finite automaton (*NFA*) is a 5-tuple, $(Q, \Sigma, \Delta, q_0, F)$, consisting of

- a finite set of states Q,
- a finite set of input symbols Σ,
- a transition function $\Delta : Q \times \Sigma \to P(Q)$,
- an initial state $q_0 \in Q$,
- a set of states F distinguished as accepting (or final) states $F \subseteq Q$.

NFA for $a^* + (ab)^*$:



REG is the class of languages recognised by a finite automaton.

An ω-automaton is a quintuple $(Q, \Sigma, \delta, q_I, Acc)$, where

- Q is a finite set of states,
- Σ is a finite alphabet,
- $\delta : Q \times \Sigma \rightarrow P(Q)$ is the state transition function,
- $q_I \in Q$ is the initial state,
- Acc is the acceptance component.

In a deterministic ω-automaton, a transition function $\delta : Q \times \Sigma \rightarrow Q$ is used.

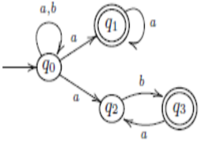
Let $A = (Q, \Sigma, \delta, q_I, Acc)$ be an ω-automaton. A run of A on an ω-word $\alpha = a_1 a_2 ... \in \Sigma^\omega$ is an infinite state sequence $\rho = \rho(0)\rho(1)\rho(2)... \in Q^\omega$, such that the following conditions hold:

1. $\rho(0) = q_I$
2. $\rho(i) \in \delta(\rho(i{-}1), a_i)$ for $i \geqslant 1$ if A is nondeterministic,
   $\rho(i) = \delta(\rho(i{-}1), a_i)$ for $i \geqslant 1$ if A is deterministic.

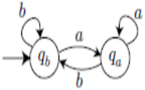
For a run ρ of an ω-automaton, let $Inf(\rho) = \{q \in Q : \forall i \exists j > i\, \rho(j) = q\}$.

An ω-automaton $A = (Q, \Sigma, \delta, q_I, Acc)$ is called

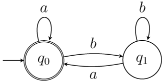
- **Büchi** automaton if $Acc = F \subseteq Q$ and the acceptance condition is the following: A word $\alpha \in \Sigma^\omega$ is accepted by A iff there exists a run ρ of A on α satisfying the condition: $Inf(\rho) \cap F \neq \emptyset$.
- **Muller** automaton if $Acc = \mathcal{F} \subseteq P(Q)$ and the acceptance condition is the following: A word $\alpha \in \Sigma^\omega$ is accepted by A iff there exists a run ρ of A on α satisfying the condition: $Inf(\rho) \in F$.
- **Rabin** automaton if $Acc = \{(E_1, F_1), ..., (E_k, F_k)\}$, with $E_i, F_i \subseteq Q$, $1 \leqslant i \leqslant k$, and the acceptance condition is the following: A word $\alpha \in \Sigma^\omega$ is accepted by A iff there exists a run ρ of A on α satisfying the condition: $\exists (E, F) \in Acc(Inf(\rho) \cap E = \emptyset) \wedge (Inf(\rho) \cap F \neq \emptyset)$.
- **Streett** automaton if $Acc = \{(E_1, F_1), ..., (E_k, F_k)\}$, with $E_i, F_i \subseteq Q$, $1 \leqslant i \leqslant k$, and the acceptance condition is the following: A word $\alpha \in \Sigma^\omega$ is accepted by A iff there exists a run ρ of A on α satisfying the condition: $\forall (E, F) \in Acc(Inf(\rho) \cap E \neq \emptyset) \vee (Inf(\rho) \cap F = \emptyset)$ $\big($or $\forall (E, F) \in Acc(Inf(\rho) \cap F \neq \emptyset) \rightarrow (Inf(\rho) \cap E \neq \emptyset)\big)$.



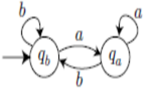
Büchi automaton for $(a+b)^*a^\omega + (a+b)^*(ab)^\omega$ with $F = \{q_1, q_3\}$



Muller automaton for $(a+b)^*a^\omega + (a+b)^*b^\omega$ with $\mathcal{F} = \{\{q_a\}, \{q_b\}\}$



Rabin automaton for $(a+b)^*a^\omega$ with $Acc = \{(\{q_1\}, \{q_0\})\}$



Streett automaton with $Acc = \{(\{q_b\}, \{q_a\})\}$.
Each word in the accepted language contains infinitely many a's only if it contains infinitely many b's (or equivalently they have finitely many a's or infinitely many b's).